

# Spender Saver Model

Ec 704 · Spring 2026 · Prof. Adam M. Guren

- Households are of two types:  $\lambda$  rule of thumb and  $1 - \lambda$  optimizing.
- For simplicity ignore money market, but we could always put it in.
- Optimizing households:

$$\begin{aligned} \max_{C_t^o, N_t^o, B_t^o, M_t^o} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}^o)^{1-\gamma}}{1-\gamma} - \chi \frac{(N_{t+s}^o)^{1+\varphi}}{1+\varphi} \right) \right\} \\ \text{s.t. } C_t^o = \frac{W_t}{P_t} N_t^o - \frac{B_t^o - Q_{t-1} B_{t-1}^o}{P_t} + T R_t^o + P R_t^o - T_t^o \end{aligned}$$

– FOCs:

$$\begin{aligned} \frac{W_t}{P_t} &= \chi (N_t^o)^\varphi (C_t^o)^\gamma \\ 1 &= \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{(C_{t+1}^o)^{-\gamma}}{(C_t^o)^{-\gamma}} \right\} = E_t \{ \Lambda_{t,t+1} R_{t+1} \} \end{aligned}$$

- Non-optimizing households:

$$\begin{aligned} \max_{N_t} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}^r)^{1-\gamma}}{1-\gamma} - \chi \frac{(N_{t+s}^r)^{1+\varphi}}{1+\varphi} \right) \right\} \\ \text{s.t. } P_t C_t^r = W_t N_t^r - P_t T_t^r \end{aligned}$$

with FOC:

$$\frac{W_t}{P_t} = \chi (N_t^r)^\varphi (C_t^r)^\gamma$$

- Household aggregation:

$$\begin{aligned} C_t &= \lambda C_t^r + (1 - \lambda) C_t^o \\ N_t &= \lambda N_t^r + (1 - \lambda) N_t^o \end{aligned}$$

- Firm side is exactly as before

$$\begin{aligned} P_t &= [\theta P_{t-1}^{1-\varepsilon} + (1 - \theta) P_t^{*1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \\ P_t^* &= (1 + \mu) E_t \left\{ \sum_{s=0}^{\infty} \frac{\theta^s \Lambda_{t,t+s}^n P_{t+s}^\varepsilon Y_{t+s}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k}^n P_{t+k}^\varepsilon Y_{t+k}} \frac{W_{t+s}}{A_{t+s}} \right\} \\ Y_t &= C_t + G_t \\ Y_t &= A_t N_t \left[ \int_0^1 \left( \frac{N_t(i)}{N_t} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

- Monetary policy is:

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_t$$

- Fiscal policy:

- Government budget constraint is

$$P_t T_t + B_t = Q_{t-1} B_{t-1} + P_t G_t$$

where  $T_t = \lambda T_t^r + (1 - \lambda) T_t^o$ .

- Government spending is constant at  $G$ .
- Assume that government lump sum taxes or rebates consumers equally when cost of debt changes.

- Market clearing:

$$N_t = \int_0^1 N_t(i) di$$

- Deriving the NKPC:

- For firm side,

$$\hat{\pi}_t = \lambda \hat{m}c_t + \beta E_t \{ \hat{\pi}_{t+1} \}$$

- Marginal costs are

$$\hat{m}c_t = \hat{w}_t - \hat{p}_t - \hat{a}_t$$

where

$$\hat{w}_t - \hat{p}_t = (\gamma + \varphi) \hat{y}_t - \varphi \hat{a}_t$$

so

$$\hat{m}c_t = (\gamma + \varphi) \hat{y}_t - (1 + \varphi) \hat{a}_t$$

- But we do not have technology shocks so

$$\hat{m}c_t = (\gamma + \varphi) \hat{y}_t$$

- Consequently:

$$\begin{aligned} \hat{\pi}_t &= \lambda (\gamma + \varphi) \hat{y}_t + \beta E_t \{ \hat{\pi}_{t+1} \} \\ &= \kappa \hat{y}_t + \beta E_t \{ \hat{\pi}_{t+1} \} \end{aligned}$$

- Deriving the IS curve:

- On the aggregate demand side, the Euler equation for the optimizers is:

$$\hat{c}_t^o = -\sigma \left( \hat{i}_t - E_t \{ \hat{\pi}_{t+1} \} \right) + E_t \{ \hat{c}_{t+1}^o \}$$

- The consumption of the rule of thumb agents is:

$$C_t^r = (W_t/P_t) N_t^r - T_t^r$$

log linearized and let

$$t_t = \frac{T_t - T}{Y}$$

we have:

$$\hat{c}_t^r = \frac{W N^r}{P C^r} (\hat{w}_t - \hat{p}_t + \hat{n}_t^r) - \frac{Y}{C^r} \hat{t}_t^r$$

- Assuming  $C^r = C^o = C$  which implies  $N^r = N^o = N$ , we have:

$$\begin{aligned} \hat{c}_t &= \lambda \hat{c}_t^r + (1 - \lambda) \hat{c}_t^o \\ \hat{n}_t &= \lambda \hat{n}_t^r + (1 - \lambda) \hat{n}_t^o \\ \hat{w}_t - \hat{p}_t &= \gamma \hat{c}_t + \varphi \hat{n}_t \end{aligned}$$

and defining  $\gamma_c = C/Y$  we can rewrite the equation for  $\hat{c}_t^r$  as:

$$\gamma_c \hat{c}_t^r = \frac{W N}{P Y} (\hat{w}_t - \hat{p}_t + \hat{n}_t^r) - \hat{t}_t^r$$

– But we know that

$$\hat{n}_t^r = \frac{1}{\varphi} (\hat{w}_t - \hat{p}_t - \gamma \hat{c}_t^r)$$

so

$$\left( \varphi \gamma_c + \frac{WN}{PY} \gamma \right) \hat{c}_t^r = \frac{WN}{PY} (1 + \varphi) (\hat{w}_t - \hat{p}_t) - \varphi \hat{t}_t^r$$

– But

$$\hat{w}_t - \hat{p}_t = \gamma \hat{c}_t + \varphi \hat{n}_t$$

so

$$\left( \varphi \gamma_c + \frac{WN}{PY} \gamma \right) \hat{c}_t^r = \frac{WN}{PY} (1 + \varphi) (\gamma \hat{c}_t + \varphi \hat{n}_t) - \varphi \hat{t}_t^r$$

– Finally,

$$\frac{WN}{PY} = \frac{1}{\mu}$$

so

$$(\mu \varphi \gamma_c + \gamma) \hat{c}_t^r = (1 + \varphi) (\gamma \hat{c}_t + \varphi \hat{n}_t) - \mu \varphi \hat{t}_t^r$$

or

$$\hat{c}_t^r = \frac{\gamma (1 + \varphi)}{\mu \varphi \gamma_c + \gamma} \hat{c}_t + \frac{\varphi (1 + \varphi)}{\mu \varphi \gamma_c + \gamma} \hat{n}_t - \frac{\mu \varphi}{\mu \varphi \gamma_c + \gamma} \hat{t}_t^r$$

– We have that

$$\hat{c}_t = \lambda \hat{c}_t^r + (1 - \lambda) \hat{c}_t^o$$

so

$$\begin{aligned} c_t - E_t \{c_{t+1}\} &= \lambda [c_t^r - E_t \{c_{t+1}^r\}] + (1 - \lambda) [c_t^o - E_t \{c_{t+1}^o\}] \\ &= \lambda [c_t^r - E_t \{c_{t+1}^r\}] - \sigma (1 - \lambda) (\hat{i}_t - E_t \{\hat{\pi}_{t+1}\}) \end{aligned}$$

– Plugging in the previous result gives:

$$\begin{aligned} c_t - E_t \{c_{t+1}\} &= \lambda [c_t^r - E_t \{c_{t+1}^r\}] - \sigma (1 - \lambda) (\hat{i}_t - E_t \{\hat{\pi}_{t+1}\}) \\ &= \lambda \left[ -E_t \left\{ \frac{\gamma (1 + \varphi)}{\mu \varphi \gamma_c + \gamma} \hat{c}_{t+1} + \frac{\varphi (1 + \varphi)}{\mu \varphi \gamma_c + \gamma} \hat{n}_{t+1} - \frac{\mu \varphi}{\mu \varphi \gamma_c + \gamma} \hat{t}_{t+1}^r \right\} \right] \\ &\quad - \sigma (1 - \lambda) (\hat{i}_t - E_t \{\hat{\pi}_{t+1}\}) \\ \left( \frac{\mu \varphi \gamma_c + \gamma - \lambda \gamma (1 + \varphi)}{\mu \varphi \gamma_c + \gamma} \right) (\hat{c}_t - E_t \{\hat{c}_{t+1}\}) &= -\lambda \frac{\varphi (1 + \varphi)}{\mu \varphi \gamma_c + \gamma} E_t \{\Delta \hat{n}_{t+1}\} + \frac{\lambda \mu \varphi}{\mu \varphi \gamma_c + \gamma} E_t \{\Delta \hat{t}_{t+1}^r\} \\ &\quad - \sigma (1 - \lambda) (\hat{i}_t - E_t \{\hat{\pi}_{t+1}\}) \end{aligned}$$

where  $\Delta \hat{n}_{t+1} = E_t \{\hat{n}_{t+1}\} - \hat{n}_t$  and  $\Delta \hat{t}_{t+1}^r = E_t \{\hat{t}_{t+1}^r\} - \hat{t}_t^r$ .

– Letting  $\Gamma = (\mu \varphi \gamma_c + \gamma - \lambda \gamma (1 + \varphi))^{-1}$ ,

$$\begin{aligned} \hat{c}_t - E_t \{\hat{c}_{t+1}\} &= -\lambda \Gamma \varphi (1 + \varphi) E_t \{\Delta \hat{n}_{t+1}\} + \lambda \mu \varphi \Gamma E_t \{\Delta \hat{t}_{t+1}^r\} \\ &\quad - \sigma (1 - \lambda) \Gamma (\mu \varphi \gamma_c + \gamma) (\hat{i}_t - E_t \{\hat{\pi}_{t+1}\}) \end{aligned}$$

or equivalently

$$\hat{c}_t = E_t \{\hat{c}_{t+1}\} - \tilde{\sigma} (\hat{i}_t - E_t \{\hat{\pi}_{t+1}\}) - \Theta_n E_t \{\Delta \hat{n}_{t+1}\} + \Theta_\tau E_t \{\Delta \hat{t}_{t+1}^r\}$$

where

$$\begin{aligned}\tilde{\sigma} &= \sigma (1 - \lambda) \Gamma (\mu \varphi \gamma_c + \gamma) \\ \Theta_n &= \lambda \Gamma \varphi (1 + \varphi) \\ \Theta_\tau &= \lambda \mu \varphi \Gamma\end{aligned}$$

which is a dynamic IS curve!

- Government budget constraint:

- Linearize

$$P_t T_t + B_t = Q_{t-1} B_{t-1} + P_t G_t$$

where  $T_t = \lambda T_t^r + (1 - \lambda) T_t^o$ .

- Rewrite as:

$$P_t T_t + \frac{\bar{B}_t}{Q_t} = \bar{B}_{t-1} + P_t G_t$$

- Let  $t_t = \frac{T_t - T}{Y}$  and linearize around a steady state with debt  $B$  and a balanced budget. Let  $\gamma_b = \frac{B}{PG+B}$ . Then with no government spending shock:

$$(1 - \gamma_b) \hat{t}_t = \gamma_b (\hat{i}_t - \hat{b}_t) + \gamma_b \hat{b}_{t+1}$$

- However, we always have zero debt and lump sum rebate so:

$$\hat{t}_t = \frac{\gamma_b}{1 - \gamma_b} \hat{i}_t$$

- Also given our assumption that it is rebated proportionally,

$$\hat{t}_t^r = \hat{t}_t$$

and so expansionary monetary policy increases transfers.

- Market clearing

$$\hat{y} = \gamma_c \hat{c}_t$$

- The full equilibrium system:

$$\begin{aligned}\hat{c}_t &= E_t \{\hat{c}_{t+1}\} - \tilde{\sigma} \left( \hat{i}_t - E_t \{\hat{\pi}_{t+1}\} \right) - \Theta_n E_t \{\Delta \hat{n}_{t+1}\} + \Theta_\tau E_t \{\Delta \hat{t}_{t+1}^r\} \\ \hat{\pi}_t &= \kappa \hat{y}_t + \beta E_t \{\hat{\pi}_{t+1}\} \\ \hat{y}_t &= \hat{n}_t \\ \hat{y}_t &= \gamma_c \hat{c}_t \\ \hat{t}_t &= \frac{\gamma_b}{1 - \gamma_b} \hat{i}_t \\ \hat{i}_t &= \phi_\pi \hat{\pi}_t + \hat{v}_t \\ \hat{v}_t &= \rho_v \hat{v}_{t-1} + \varepsilon_t\end{aligned}$$

- This can be simplified, by combining everything into an AS and AD equation, but I will just load it into Dynare as is.